Johann Sebastian Bach RICERCAR Musikalisches Opfer 1
INTONATION by Marc Sabat and Wolfgang von Schweinitz (2001)

an experimental application of Extended Rational Tuning

P L A I N S O U N D   M U S I C   E D I T I O N

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ABSTRACT

This piece is part of an ongoing research and compositional project initiated by Marc Sabat and Wolfgang von Schweinitz involving the analysis and re-notation of music for performance in Just Intonation. The scores are annotated with accidentals using a fully transposing enharmonic notation system that we have developed. The goal of this text is to introduce the general principles underlying this work, its motivations, as well as discussion of some of the possibilities we believe it offers for both interpreters and composers.

INTRODUCTION

The chromatic notation of pitch employed in Western music has been used to represent many historical systems of intonation and temperament. This convention currently accepts a theoretical division of the octave into twelve equal semitones, understood to represent fixed, distinct points of reference, and applied with varying degrees of intonational tolerance depending on the means of sound production and musical context. Nonetheless, it is clear from the various possible spellings of enharmonically similar tones (i.e. Bb and A#) that the twelve-tone model has not always been considered sufficient. In general, the realization of notated pitches has often been understood to be approximate, subject to variations which may result from common practice or subjective taste and style, but which are equally often indeterminate results of inharmonicity or inaccuracy of realization.

In the twentieth century, with the advent of twelve-tone composition, the precise realization of mathematically exact Equal Temperament was advocated as a new intonational ideal, and has become realizable through the capabilities of digital sound representation and synthesis. At the same time, electronic and digital technology has enabled sounds to be abstractly represented as mathematical functions in a continuous domain with acoustical parameters frequency, phase, amplitude, time, correlating to various perceptual equivalents (pitch, timbre, volume, morphology, tone, character, shape, duration, attack, etc.). The openness of this new model has led to a remarkably rich body of experimental music investigating the capacity of instruments to produce new (often unpitched) sounds.

This expanded sound-field has also inspired consideration of the precise relations between its elementary consituents, and in particular has allowed a re-examination of the notion of “harmony” as a perceptual phenomenon of the relationship between pitched sounds. James Tenney’s paper “John Cage and the Theory of Harmony” (1983) sets out some initial principles by which such investigations might begin, as part of the development of a more general theory of musical perception.

Given a continuous frequency domain, bounded only by the limits of hearing, what are the underlying principles by which specific decisions about intonation can be made? What possibilities exist in a given situation, how do they sound, and in what way do they affect our perception of music? What kind of notation can indicate these variations in a simple and intuitive manner?

In describing the perceived relationship of two pitched sounds, it is possible to compare which tone is lower and which is higher (melodic distance). A second measure of the distance between tones is determined by the complexity of the interval they form (harmonic distance). In this paper, the term rational interval will be used when pitches are exactly related by whole-number ratios. Such intervals have the characteristic of producing periodically repeating patterns of sound that have distinct effects on our perception. In general terms, harmonic distance between pitches can described by comparing the ratio of their fundamental frequencies to the nearest, simplest whole-number ratios. (For example, the irrational Equal Tempered minor third is very close to two rational intervals, 27:32 and 16:19.)

Classical Greek music theory describes the development of pitch sets based on both of these properties. Ptolemy recounts the derivation of various melodic modes by applying rational proportions to the division of a string, emphasizing the principle of harmonic distance, while Aristoxenus proposes adding various sizes of melodic steps (whole-tone, semitone, quarter-tone) in a method similar to modern tempered systems. Each approach is based on psychoacoustic properties of our pitch perception process. The ability to readily assess simple periodic relationships is related to the process of filtering and distinguishing timbres in a mass of sounds, and the tendency to roughly accept “semi-” tones as distinct pitch-classes is related to the average critical bandwidth of our ear's nervous mechanism.

Equal Temperament provides an efficient means of measuring melodic distance, but its ability to precisely describe intervals is compromised by the fact that it is a temperament, i.e. a deliberate mistuning of certain rational intervals designed to produce a finite set of fixed pitches. Of course it is possible to notate intervals more accurately by using finer equidistant gradations. For example, Alexander Ellis proposed the system of “cents” to indicate deviations from the Equal Tempered tones in steps of 1/100 of a semitone. Such a method allows for a tolerance of error that is generally finer than the natural fluctuation of most acoustically produced sounds. The main disadvantage of this method of notation is that it does not intuitively correlate to our sense of harmonic perception (for example, a pure fifth above any Equal Tempered pitch would require that +2 be written over it to represent 702 cents, which approximates the frequency ratio 2:3). It would seem more logical to have a simple notation for a simple harmonic distance.

Imagine a string quartet tuning the open strings to C-G-D-A-E in perfect fifths. If the A-string registers “in tune” on an (Equal Tempered) electronic tuner, then the other strings will deviate (in cents) as follows: C (-6); G (-4); D (-2); E (+2). It is well known that the open E-string of the violins forms a relatively complex-sounding interval with the open C-string of the cello. In fact, the open E (+2) beats against the open C’s tenth partial, a different ‘E’ which is 21.5 cents lower (one Syntonic Comma). If A is tuned to 440 Hz, then the open E string is 660 Hz, while the E which is the tenth partial of the open C string is approximately 651.85 Hz, producing beating between the two fundamentals at 8.15 Hz.

To achieve a soft consonance by eliminating the beats between C and E, in earlier historical practice the fifths would often each be compressed by 1/4 of a Syntonic Comma (1/4-Comma Meantone Temperament). Variations of this process led to the development of many different flavours of temperament, including the unequally-spaced “Well-Tempered” systems explored in Bach’s time, in which different keys had different sounds based on their detonation.

An alternative solution would be to keep the fifths pure, acknowledge the existence of two enharmonic E’s one comma apart, and notate the appropriate one based on context. This kind of adjustment is certainly familiar to string quartet players, and such decisions are often made unconsciously. It is our intention to provide tools for notation, analysis and theoretical understanding of this process, allowing intonation to be based on a broader palette of precisely-tuned sonorities.

In our own work, we propose a fully transposing enharmonic notation to specify harmonic distance and distinguish between similar tones derived from different rational intervals. It allows for the distinct notation of composite periodic sounds, and offers the possibility of notating intonation exactly.

Pitched sounds are in themselves periodic, and when two (or more) periodic sounds have a rational relationship to each other (like overtones), they form a composite periodic sound. Such sonorities form the basis of an intuitively realized “clean intonation”. Simple periodic sounds are smooth, fusing together, reinforcing each other and giving the impression of being “in tune”. Complex periodic sounds can be smooth or rough, consonant or dissonant, but all share the property of producing a distinctive sound pattern or “signature”, possibly because they are in fact very fast polyrhythmic patterns. The ability of the ear to perceive these signatures is the basis of our perception of harmonic relationships.

It is our contention that by investigating the properties of periodic sounds, it is possible to further develop musical understanding of the phenomena of harmonic perception, and conversely, that a contemporary investigation of harmony demands a more precise differentiation of sonorities.

INTERPRETATION OF A COMPOSITION IN DIFFERENT TUNING SYSTEMS

Contemporary string instruments (violin, viola, cello, bass) are uniquely equipped to realize different intonational possibilities. They are unrestricted by frets. The consistency of modern synthetic string materials allows the open strings and their harmonics to provide a relatively stable frame of reference. The ability to play simultaneous sounds allows a player to readily construct and check different tunings. Most importantly, the timbre of string instruments is particularly sensitive to the effects of intonation, especially in the production of difference tones and reinforced common partials, both of which are readily perceived characteristics of composite periodic sounds.

It is due to these qualities that our first experiments in analyzing and notating an intonation interpretation involve string instruments. A decision was made to examine Johann Sebastian Bach's *Ricercar a 3* from *Musikalisches Opfer*, BWV 1079, and to compose a tuned version for string trio. The especially chromatic character of the music, as well as the extent and variety of tonal modulations make this piece a particularly challenging test case for applying Just Intonation.

In approaching the question of how to tune any music one has two options - either every written note is assumed to represent a fixed pitch, or it is permitted to have different tunings depending on context, in which case it may also be necessary to clarify these variations by means of additional notation. All of the traditional keyboard temperaments (including Equal Temperament) as well as the Pythagorean Tuning fall into the first category. Depending on the chosen system, enharmonically similar pitches (i.e. Bb and A#) may have different tunings, but in each of these systems no additional re-notation of the original score is necessary. They only require that the interpreter have a clear understanding of how each written pitch is to be tuned.

In examining this first class of possible tunings, it is clear that accurate realization of keyboard temperaments on string instruments is extremely difficult. To construct an interval by ear, a keyboard tuner generally seeks to eliminate beats between common partials and produce a smooth, stable periodic sound, based on a familiar signature. If the desired interval is to be tempered, various means are employed, such as measuring the speed of beats in specific registers and comparing the detuning of similar intervals. All of these methods demand a kind of slow, careful analysis of the sound that cannot be expected to consistently take place during the course of a performance. At best, players could rehearse slowly with a keyboard instrument or an electronic tuner as reference, and train themselves to approximate as accurately as possible.

1/4-comma Meantone Temperament has one advantage among these keyboard-based systems - major thirds are to be played pure, and can therefore be tuned by ear. Once the open strings have been adjusted to narrow tempered fifths, one can theoretically construct all other tones in this system by ear, using pure major thirds above and below each of the open strings. Still, playing tempered pitches accurately without any fixed reference (frets or keyboard) is limited by our ability to 'memorize' the detuned intervals, and cannot be easily verified by the ear alone.

The Pythagorean Tuning, which has commonly been used to build melodic scales in many musical traditions (i.e. Chinese, Arabic, Persian), has the property of being readily constructed entirely by ear. The fifths are tuned pure, and then every other tone is determined from the open strings by a chain of pure fifths and fourths.

In spite of its relatively complex harmonic intervals (for example, the major third between open C and open E mentioned above), this system remains in common use especially for solo melodic music and is known to some violinists as “expressive intonation” (narrow semitones, high leading tones, low minor thirds, higher ‘sharps’, lower ‘flats’). (A notable recent composition using this tuning is John Cage’s Cheap Imitation for solo violin.) Nevertheless, accurately producing the more complex Pythagorean interval classes (i.e. minor second, major third, minor sixth, major seventh) still poses difficulties similar to the tempered systems.

In all of the above-noted one-to-one tuning systems, players must recognize which intervals can be tuned by ear, and then melodically determine the remaining notes as accurately as possible (perhaps with the aid of a tuner). In the second class of possible intonations, a written note is tuned based on its context, rather than representing a fixed pitch. In other words, a written Bb might be tuned in several different ways during the course of a composition. Ptolemaic Tuning, also known as 5-limit Just Intonation, is the simplest such example. It is an extension of Pythagorean Tuning, constructed by forming a two-dimensional lattice of pure fifths and pure thirds, and thus providing an interlocking network of pure major and minor triads.

As an immediate logical consequence of introducing new tones, the potential realization of melodic structures becomes more complex. For example, in Pythagorean Tuning there is only one whole-tone (8:9). Two successive whole-tones make a ditone (64:81). In other words, these two simple melodic steps produce a Pythagorean major third which is harmonically more complex than the soft-sounding pure major third (4:5). The Meantone Temperament starts with this pure third, and divides it into two equal ‘meantones’ which are tempered intervals, smaller in size than a 8:9 whole-tone, but larger in size than the ratio 9:10.

In the Ptolemaic Tuning, this pure major third can be melodically divided into two unequal whole-tones (8:9 and 9:10). In the Ptolemaic C Major scale, for example, the progression C-D-E is usually played as 8:9 followed by 9:10, mirroring the overtones 8:9:10 over C. In the same scale, the progression G-A-B is played as 9:10 followed by 8:9 (so that A is tuned as a pure third above the subdominant F). Thus, tuning harmonically simpler sonorities leads to increased melodic complexity.

The application of Ptolemaic Tuning, by introducing variations in the way notes are tuned, suggests the necessity of refining the notation of pitches in the written score. In C Major, for instance, the notes E, A, B are all tuned as 4:5 thirds over C, F, G. These thirds are all lower than the respective Pythagorean pitches by a Syntonic Comma. In our notation, we indicate this alteration by an arrow attached to the accidental in question. Thus, the major third in C Major is E-natural-*arrow-down*. A similar logic operates in reverse - for example, in C minor, the Eb is a pure major third below G, and thus it is written as Eb-*arrow-up*. This procedure is based on a pitchletter-notation developed in the 19th Century by Hermann von Helmholtz as part of his studies of tuning and acoustics, which are documented in *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (1862).

American composer Ben Johnston, one of the pioneers of Extended Just Intonation, developed an alternative notation which is based on the Ptolemaic C Major scale. In his system, the seven natural 'white' notes C D E F G A B, without accidentals, are understood to be tuned in Just Intonation as described above. In this approach, the notation D : A is understood to represent the interval 27:40, which is one Syntonic Comma less than the perfect fifth 2:3 (written D : A+). As a result of this decision, the Classical notation of the series of fifths is interrupted, with the consequence that transposed melodic structures do not always maintain similar notational forms. For example, the whole tone 8:9 might be written C : D or G : A+. While we understand Johnston's decision to found his notation on Harry Partch's premise of 'Monophony', we believe that a generally acceptable reform of musical notation must embrace the principle of symmetrical transposability. Consequently, we have chosen to base our system on the various 19th Century models in which the series of fifths is traditionally represented by flats, naturals, and sharps.

Given multiple possibilities for each written tone, how does one make choices? A first attempt might be to seek out the simplest (most 'consonant') solution for each vertical sonority. However, even in the simplest triadic progressions, conceptual difficulties arise. Take, for example, a progression such as I-IV-ii-V-I in C Major. The first two triads pose no problems - first the E and then the A are tuned one comma lower (as pure major thirds) and C is maintained as a common tone. The last two triads, by symmetry, operate similarly, with G as a common tone.

Now, in analyzing the tuning of ii (d minor triad), observe that the notes F and A-arrow-down are already sounding in the IV chord. Thus, we could keep these notes common and add a D-arrow-down as root to produce a pure triad. Meanwhile, working backwards, since I and V have G in common, the logical tuning for V would be G-B-arrow-down-D. Thus, in the progression ii-V, the D would have to be retuned. Alternately, the ii triad could be tuned a comma higher, but then both F and A would have to be enharmonically altered in the progression IV-ii. If we decide to avoid retuning any notes, while maintaining pure triads for each sonority, then the last two triads in the progression would have to be one comma lower, ending up in C Major-arrow-down (comma shift).

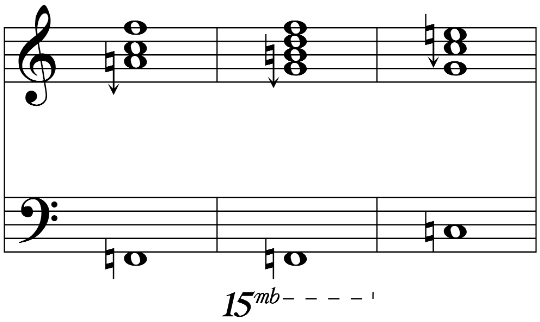
Such arguments have often been used to make the case that Just Intonation cannot be applied in practice. Instead, we propose that all of the possibilities and consequences should first be fully analyzed and tested in a musical realization. Our initial assumption that every vertical sonority be tuned as simply as possibly has led to two options - enharmonic retuning of similarly-notated tones or a comma shift.

A third possibility would be to consider alternate (more complex) tunings of some of the chords. For example, in the progression above, if the triads on IV and ii are played with Pythagorean thirds, then no melodic retunings are necessary. Such a solution might be considered if all common tones are to be sustained without enharmonic alterations and a comma-modulation is not desirable. Yet another alternative in this manner would be to follow Helmholtz's proposal to tune the ii triad based on the C Major scale tones, as a dissonant triad D-F-A-arrow-down (with the fifth tuned to the ratio 27:40). In certain progressions, notably with the ii seventh chord in first inversion, where the root functions as a dissonant added sixth over the IV triad, anticipating V, this tuning (16:20:24:27) is of significant interest.

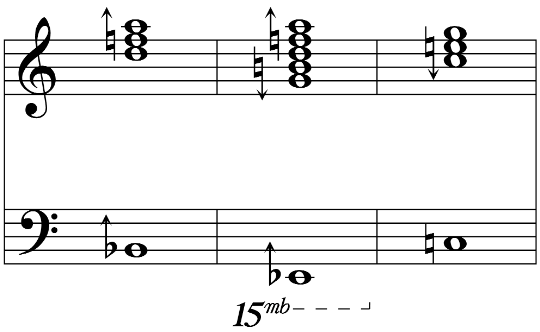
In general, this kind of analysis re-examines the relationship between melody and harmony, making it possible to imagine a melodically-articulated tonality. In other words, at any given point in the music, the set of available pitches can be based on one (or more) specific harmonic field(s). As the music modulates, new tones are introduced. Consider again the scale of C Major in Ptolemaic Tuning. The progression G-A-B is tuned G-A-arrow-down-B-arrow-down. If the same three notes were to be played in G Major, they would be tuned G-A-B-arrow-down. The A would therefore have to be played a comma higher than in C Major. In this approach to tuning, a melodic interpretation is determined directly by the harmonic logic of the music, and conversely, the intonation of the melodies reveals an implied harmony.

Ptolemaic Tuning, with its pure fifths and thirds, offers a relatively familiar-sounding model for well-tuned interpretation of triadic tonal music, although it still demands a number of distinctly new interpretative techniques (as noted in the progression discussed above). However, as more complex chords come into consideration, there are tuning variations that cannot be realized within the Ptolemaic system. For example, consider possible tunings for a Dominant Seventh chord based on the pure major triad 4:5:6.

The Pythagorean minor seventh 9:16 is constructed from two adjunct perfect fourths. Superimposing this interval over the root of a major triad yields ratios 36:45:54:64. Using this tuning in the progression IV-V7-I, the root of IV and the seventh of V7 are tuned to the same pitch, as expected. The fundamental of the generating harmonic series for this tuning is the same pitch as the seventh of the chord (its 64th partial), transposed down seven octaves. In other words, if the chord's root were G, then the chord tones would be tuned as overtones of a low F.



Jean-Philippe Rameau (1683-1764), in his *Traité de l'Harmonie*, suggests another possibility. He tunes the chord by stacking successive pure thirds (4:5, 5:6, 5:6), producing the ratios 20:25:30:36. In the major-key progression ii-V9-I, by using this tuning the tones of ii can be sustained over the Dominant Ninth chord without retuning. The fundamental of the harmonic series producing these tones would be a pure major third and four octaves below the root of the chord (its 20th partial). Thus, if the root were once again G, then the pitches would be tuned as overtones of a low Eb-arrow-up.



Yet another solution is to follow the progression of pitches in the third octave of the overtone series - 4:5:6:7. In this case, the fundamental is the same pitch-class as the root, two octaves lower. An empirical comparison of sonority alone suggests that this septimal Dominant Seventh is of significant musical interest. However, by its inclusion of the seventh partial, which falls outside the Ptolemaic Tuning, this chord requires an extended tuning system including intervals produced by higher partials.

Violinist Giuseppe Tartini (1692-1770), whose playing technique included producing strong bass-register 'difference tones' by means of Just Intonation, advocated use of the seventh partial and reputedly pioneered a sign for this overtone - a mirrored and inverted '7' - which we have incorporated in our notation. This symbol came to our attention through the publications of German musicologist and Just Intonation advocate Martin Vogel.

By analogy, examining variations in the possible tuning of sounds leads to the investigation of various other prime partials as potential material for an Extended Rational Tuning based on Just Intervals. The complexity of such a system should be bounded primarily by our ability to hear differences of detail in the sounds. Ideally, it would provide a means to more accurately distinguish and notate harmonic phenomena, and offer the possibility to consider sonorities based on their perceptibility, rather than by means of purely theoretical, aesthetic or historical grounds.

In the case of *Johann Sebastian Bach RICERCAR Musikalisches Opfer 1*, we eventually were led to include for consideration intervals produced by the prime partials 2, 3, 5, 7, 17 and 19, determined by situations presented in the composition's harmonic and melodic musical structures. The omitted “quarter-tone” prime partials 11 and 13, while providing rich material for new compositions, fall outside the tonal framework of Bach’s music in our opinion.

One of the most notable properties supporting the use of the septimal Dominant Seventh is that its difference tones reinforce the harmony of the chord - in particular, the root of the chord is identical to the fundamental of the generating harmonic series. By extending this logic, it is possible to tune a Dominant Ninth Chord as 4:5:6:7:9, maintaining similar psychoacoustic reinforcement of the sonority. To similarly construct a Dominant Flat Ninth Chord, the simplest overtone from the same series would be 17, dividing the whole tone 8:9 into two unequal ratios 16:17 and 17:18. Thus the chord would be 8:10:12:14:17, which also suggests one possible tuning for the Diminished Seventh Chord (10:12:14:17).

A similar harmonic argument can be proposed for intervals produced by the 19th partial. Consider the minor triad 16:19:24 produced in the fifth octave of the harmonic series. Notably, the first-order difference tones produced are the overtones 3 (16:19), 5 (19:24), and 8 (16:24), reinforcing both the root and fifth of the triad. Such a tuning closely resembles both the Pythagorean and Equal Tempered minor triads, but remains distinct from either sonority, due to its relatively simple periodic structure.

The melodic sequence 16:17:18:19 falls within small (less than 5 cents) tolerances of Equal Tempered semitones. In fact, Vincenzo Galilei (ca. 1525-1591) proposed 17:18 as a useful ratio for constructing Equal Tempered frets for the lute. As a Just Intonation model for chromatic melodic lines, it can provide a useful alternative to the more unequal steps found in Ptolemaic Tuning or the 7-Limit Tartini Tuning.

In response to these musical applications of the 17th and 19th partials, which were confirmed by testing the sounds in practice, our intonation *Johann Sebastian Bach RICERCAR Musikalisches Opfer 1* is based on a 19-Limit Extended Rational Tuning.

As consideration passes to increasingly higher prime partials, the ability of our perception to distinctly distinguish them from similar composite intervals rapidly diminishes. Nevertheless, they remain subjects of musical exploration, especially since such tones (which can be accurately reproduced by computers) are produced by simpler intervals as summation tones.

In our notation, we have developed exact signs up to the 36th overtone of any fundamental. Higher prime overtones are currently easily notated approximately (with an error tolerance of about 2 cents) until such time as an exact notation should prove necessary in any particular case. Higher composite overtones with prime factors less than 36 can all be written precisely.

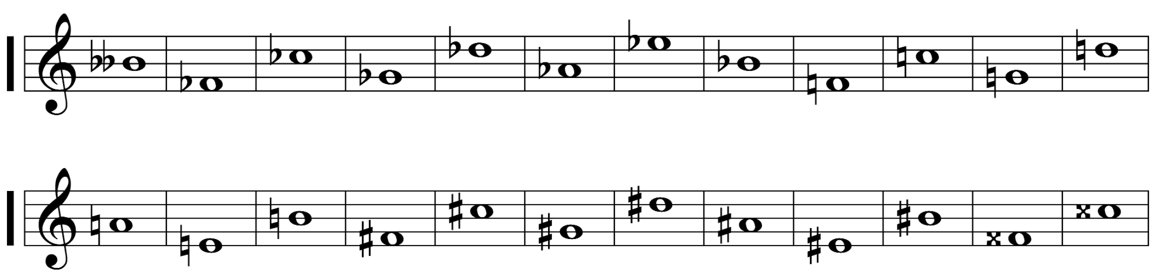
ANALYSIS AND INTONATION OF THE RICERCAR

To make actual decisions about the intonation of Bach's Ricercar, the first stage was to sketch an analysis of the score in Ptolemaic Tuning. This version was tested, in part, on a two-manual harpsichord using Hermann von Helmholtz's system of temperament, in which the fifths and thirds can be very accurately represented on a keyboard instrument. It became clear that applying Ptolemaic tuning in a consequent manner produced certain situations that we found musically unsatisfactory for Bach's composition. This conclusion led us to a second stage of analysis, gradually incorporating tunings based on the higher partials mentioned above (7, 17, 19).

(While a complete description of Helmholtz Temperament falls outside the scope of this paper, it can be briefly outlined as follows. Taking our earlier example of the string quartet's open strings, consider the four fifths C-G-D-A-***E***, producing a Pythagorean Third between C and E. Now, alternately, it is possible to start from C and tune eight fifths in the downward direction: C-F-Bb-Eb-Ab-Db-Gb-Cb-***Fb***. This final Fb is lower than the open E string by a Pythagorean Comma. Coincidently, this comma is only 2 cents greater than the Syntonic Comma we would require to obtain E-arrow-down, the pure third over C. By dividing this 2 cent "Schisma" across the 8 fifths, making each smaller by about 0.2442 cents, which is arguably insignificant for acoustic sounds, the resulting Fb would become identical to the E-arrow-down we require. The Helmholtz Temperament makes all fifths smaller by this amount, producing a system of tones in which almost-pure fifths and exactly-pure major thirds can be played on a keyboard instrument.)

Having decided to experimentally investigate more complex rational intervals, we determined to develop accurate computer-based tools to empirically test the new possibilities as they arose. For this purpose, a software sampler with very accurate microtuning options was programmed, using a simple reed-organ sound. The use of a transposed single-cycle waveform sample with a consistent spectrum enabled us to reliably assess tuning decisions and to judge the precise timbral consequences of a particular intonation.

The particular software used (Unity DS-1) allows the exact tuning of each MIDI note-number to be specified in cents, accurate to four decimal places. Since each MIDI patch can only play 12 notes per octave, we decided to devise a set of different patches with various tunings we needed. First, 2 patches were dedicated to producing 24 notes from the Pythagorean series of perfect fifths:



These were transposed by various commas to produce additional patches. For example, if a note needed to be tuned as a 17th partial over F, we would choose the pitch Gb from PATCH 1(17). Using music notation software (Finale), we could assign the patches to different staves, and place notes in appropriate staves according to their tuning by copying and pasting, thus allowing us to readily compare different variations. On three additional staves, the actual playing score was notated using a custom font (Enharmonic) that we designed for our system of accidentals. This method allowed us to construct and consider the sound of different intonations.

The only restriction in our procedure was to limit the possibilities considered to strictly periodic sounds. Although our judgment would often lead us to unorthodox and complex solutions of particular musical situations, we decided to avoid “smoothing out” any difficulties by making use of tempered compromises, as we were interested to hear the effect of music made entirely from rational intervals. One consequence is a tendency to focus attention on the physical properties of the sonorities — the production of combination tones, reinforcement of common partials, and periodic beating patterns.

A complete annotated discussion of all tuning decisions made in our intonation falls outside the scope of this text. Nonetheless, as a guide to studying the score it might be of interest to examine several brief passages in detail to indicate the process by which our decisions were made. As an example, consider the tuning of the opening theme.

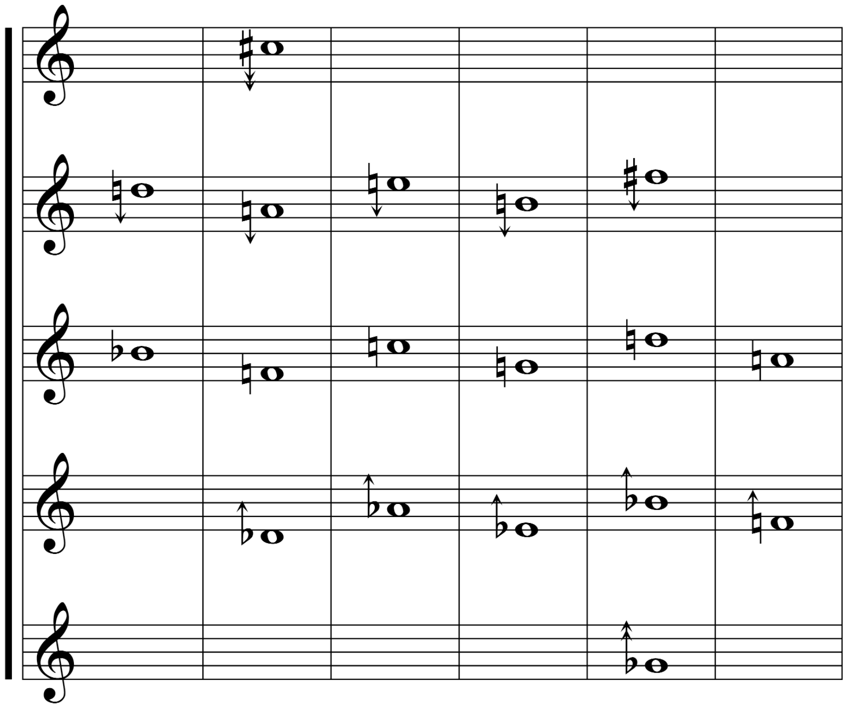
The first five notes outline primary harmonic tones from the c minor tonality, i.e. the minor triads on i and iv and the major triad on V. This is followed by a descending chromatic scale from G down to B, a diatonic descent to G, followed by another delineation of the central tonality - the pitches G, C, F - and concluding with a diatonic descent to C. As a melodic design this structure functions to harmonically establish the principal tonality, and it leads us to the first question - how to determine the tuning of a tonal harmonic field.

As a first case, let us consider the harmonic field of C in Ptolemaic Tuning. The three primary triads (C, F, G) offer one logical starting point. By tuning the pure major triad 4:5:6 above each of these three tones, nine pitches are produced: C/E-arrow-down/G; F/A-arrow-down/C; G/B-arrow-down/D. Taken in ascending order, these pitches form the diatonic C Major scale: C/D/E-arrow-down/F/G/A-arrow-down/B-arrow-down. It should be noted that classical triads formed from this set of pitches are all simple consonances with the exception of ii, which was mentioned in an earlier discussion. (Depending on the progression involved, it would be possible to leave this chord dissonant, retune the D to D-arrow-down, or retune both F and A-arrow-down upward.)

A similar logic applied to the three pure minor triads 10:12:15 based on C, F, G yields pitches C/Bb-arrow-up/Ab-arrow-up/G/F/Eb-arrow-up/D (descending melodic minor). In this case the problematic triad is VII, and can be considered similarly. (Either it is left dissonant, or the F is retuned to F-arrow-up, or the Bb-arrow-up and D are both retuned downward. In this case, another solution exists as well from the concept of

“harmonic minor” - a B-arrow-down from the major scale could be substituted for the root, producing a Ptolemaic diminished triad.)
Taking both sets together, so far 10 chromatic pitch-classes have been tuned. It remains to consider C#/Db and F#/Gb. Given the premise of triadic harmony based on pure fifths and thirds, the most obvious options would be C#-two arrows-down (as major third to the fifth A-arrow-down/E-arrow-down); Db-arrow-up (as fifth under F/Ab-arrow-up); F#-arrow-down (as third of the secondary Dominant D Major); Gb-two arrows-up (as minor third of the relative Major Eb-arrow-up). These enharmonic variations with different tunings are not unique to experimental Just Intonation - similar distinctions occur in Meantone Temperament and in historical practice many examples exist of "split-key" instruments designed exactly for this reason.

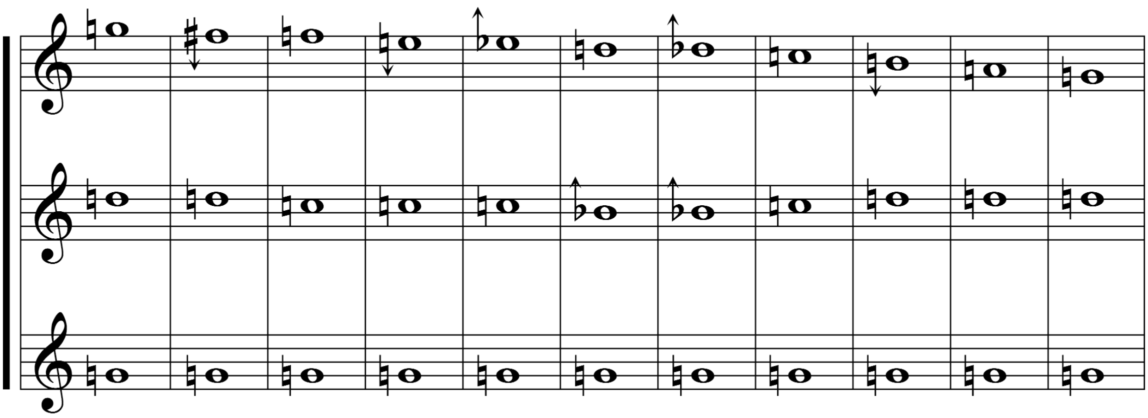
In the preceding analysis a basic 5-Limit triadic harmonic field has been outlined in the tonality of C. It can be easily diagrammed as a local subset of the 2-dimensional lattice of fifths and thirds:



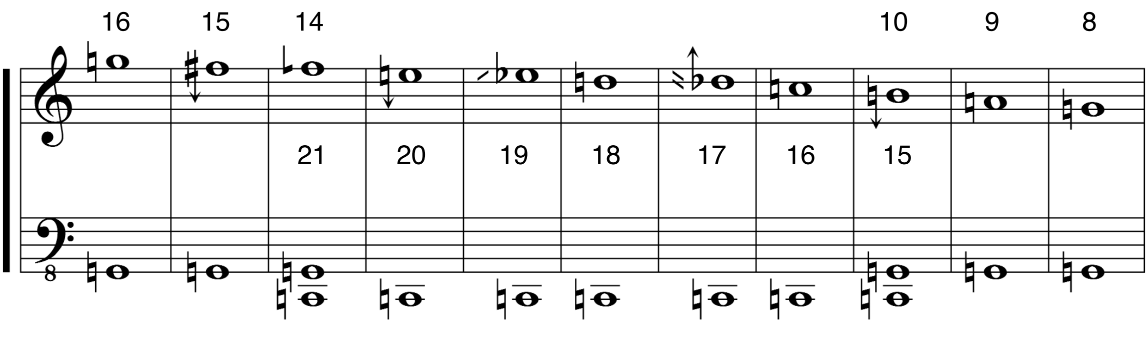
This symmetric diagram, in a form (‘Tonnetz’) pioneered by Gottfried Weber (1779-1839), represents perfect fifths (left-to-right) and pure major thirds (bottom-to-top) and contains all the tones that were mentioned in the preceding discussion. Our first version of Bach’s opening theme in Ptolemaic Tuning was based on these pitches, and remains one of the harmonically logical options. One of its characteristics is the introduction of three different semitones in the descending chromatic line. The notes G/F#-arrow-down/F/E-arrow-down/Eb-arrow-up/D/Db-arrow-up/C/B-arrow-down delineate the following sequence of downward ratios:

16:15 / 135:128 / 16:15 / 25:24 / 16:15 / 135:128 / 16:15 / 16:15

To a listener accustomed to equidistant semitones, these unequal steps produce a distinctly “microtonal” sound which at first may seem quite unusual, especially in the absence of harmonizing pitches to clarify the relationship of successive tones. The tuning immediately becomes clearer with the following sustained pitches:



Nevertheless, from a musical point of view, this theme is presented as a solo melodic line at this point in the composition, which led to the question - is there another solution which maintains harmonic sense in the chromatic line, while keeping the steps more “equal”. In keeping with our investigation of higher partials up to 19, the alternative solution would be to consider a scale of successively descending overtones which fulfills the harmonic underpinning of the music (should there be a sustained drone) yet does not require it to sound convincing. The following sketch shows our tuning, with the numbers representing overtones over the low fundamentals indicated on the left.



This solution satisfies both harmonic and melodic considerations in one tuning. It should be noted that this does not become a “universal” tuning for the theme that is applied to each occurrence. Rather, various harmonic possibilities are set into play by the musical context, and the intonation is varied accordingly to place these into relief.

As one additional example, consider how the theme is varied upon its second entry in bar 12. The F#-arrow-down in the middle voice is marked with a star, and annotated with a small enharmonic variant tuned as the seventh partial of Ab-arrow-up. These two pitches (F# and septimal Gb) differ by a small interval we have named the "Enharmonic Tritone Schisma" (224:225), which has a very distinct function in tonal modulation. Tuned as the septimal Gb-arrow-up, it continues the harmony of the preceding Ab-arrow-up fundamental. Tuned as F#-arrow-down (approximately 7.7 cents higher) it anticipates the tonal region of the (dominant function) D fundamental. It is exactly the confusion between these two similar pitches that produces the extraordinary harmonic effect of modulation that occurs here. This particular effect can be found to recur on several occasions in the score; in each case the accidental is marked with a star below it.

In earlier discussion, it was mentioned that in tuning music using composite periodic sounds, enharmonic alteration of individual notes, comma shifts (i.e. enharmonic alteration of an entire set of pitches), and complex new sonorities must all be considered. A closer examination of decisions made in the score will reveal examples of each of these possible situations. Throughout the composition, our intonational decisions were based on finding varying balances of complexity between the harmonic and melodic musical structures. At times the score follows a harmonically simple 5-Limit solution, and at other times is based on septimal harmony or on complex structures using the 17th and 19th partials.

CONCLUSION

The process of intonation analysis and realization outlined in this paper is proposed as a tool for contemporary interpretation, both as a means to understanding the application of tunings in historical performance practice and in discovering new possibilities for the realization of both modern and traditional repertoire. It is based on the premise that accurate analysis of harmonic perception and intonation can only be achieved in relation to composite periodic sounds. The methodology presented here can be applied both to simple pedagogical goals, i.e. better traditional tuning, and can also be the basis of radical new experiments in interpretation and/or composition.

One of the psychoacoustic properties affecting our perception of intonation is an ability to adjust our harmonic hearing to a given degree of mistuning (tolerance). This may be attributed to various factors, including (among others) the degree of natural inharmonicity in a particular sound, modulations of component frequencies, and properties of the critical band phenomenon, by which closely separated pitches produce effects of pleasant beating or roughness rather than distinct frequencies.

Notating precise Just Intonation is not an attempt to ignore the reality of tolerance or of the musical possibilities that have resulted from our ability to accept varying degrees of temperament. Rather, it can also be regarded as an experiment to determine how refining the degree of tolerance can provide new harmonic insights. For example, investigation of the partials 17 and 19 suggests a possible acoustic-harmonic understanding of intervals in Equal Temperament, with significant applications to the interpretation of early-20th-century 'atonal' and 'twelve-tone' music. In particular, this work allows for a redefinition of the Classical distinction between 'consonance' and 'dissonance', substituting a continuum of tuned sonorities with varying degrees of harmonic complexity.