An informal introduction to the Helmholtz-Ellis Accidentals
by Marc Sabat

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In learning to read HE accidentals, without having to rely on an electronic tuning device, it is important to be familiar with three things:

First, to keep in mind the natural tuning of intervals in a harmonic series, which deviate from the tempered system.

Second, to get to know how the accidentals refer to these overtone relationships.

Third, to observe that each written pitch may be related to many other pitches by natural intervals, and to tune it accordingly.

In most cases, this approach will allow the player to quickly and intuitively play just intonation (JI) pitches quite accurately. Any remaining adjustments can be made by ear, based on the specific sound of JI intervals.

Just intervals are readily learned because they are built up from simple, tuneable harmonic relationships. These are generally based on eliminating beating between common partials, finding common fundamentals and audible combination tones, and establishing a resonant, stable sonority which maximizes clarity: both of consonance and of dissonance.

A well-focussed JI sound is completely distinct from the irregular, fuzzy beating of tempered sounds. Just consonances, when marginally out of tune, beat slowly and sweetly and may be corrected with the most subtle adjustments of bowing or breath. Just dissonances produce a sharply pulsing regular rhythm and have very clear, distinct colors.

To become familiar with the notation and sounds of JI, the fundamental building blocks are prime number overtones 3, 5, 7, 11 and 13, each of which is associated with a specific pair of accidentals and a basic musical interval.

3 is associated with the signs flat, natural, sharp and refers to the series of untempered perfect fifths (Pythagorean intonation). Generally, A is taken as the tuning reference, and the central pitches C-G-D-A-E can be imagined as the normal tuning of the orchestral string instruments. The just C is rather lower than tempered tuning because of the pure fifths. The further this series is extended, the greater the deviation from tempered tuning: the flats are lower, the sharps higher.

5 is associated with arrows attached to the flat, natural, sharp signs and refers to the pure major third. These arrows correct the Pythagorean intervals by a Syntonic Comma, which is approximately 1/9 of a wholetone or 22 cents. So, for example, the note E-flat arrow-up is a just major third below G, and the note F-sharp arrow-down is a major third above D. In most music, flats are often raised by a comma and sharps are lowered. Because of the open string tuning, it is common to sometimes raise F and C (to match A and E) and to sometimes lower A and E (to match F and C). Corrections by one Syntonic Comma have been used throughout Western music history and are relatively familiar to the ear. However, traditionally these corrections have been hidden by players, for example in Meantone Temperament where fifths are mistuned narrow by ¼ comma so that the third C-E ends up sounding pure. More recently, the currently prevailing Equal Temperament has made us accustomed to beating thirds, so at first the pure intervals may seem unfamiliar. To play the arrows accurately, one must carefully learn the sound of the consonant major and minor thirds and sixths, and learn to articulate comma differences clearly.

7 is associated with a Tartini sign resembling the numeral. It corrects the Pythagorean intervals by a Septimal Comma, which is approximately 1/7 of a wholetone or 27 cents. When the Pythagorean minor third is lowered by this amount, it becomes a noticeably low third often heard in Blues music.

11 is associated with the quartertone signs (cross and backwards flat). The accidental is used to raise the perfect fourth by 53 cents, producing the exact tuning of the 11th partial in a harmonic series. The sound is most easily learned by playing one octave plus one fourth and raising it by a quartertone.

13 is associated with the thirdtone signs (cross and backwards flat, each with 2 verticals). The accidental is used to lower the Pythagorean major sixth by 65 cents, producing the exact tuning of the 13th partial in a harmonic series. The sound is most easily learned as a neutral-sounding sixth, one-third of the way between the just minor and just major sixths (closer to minor than to major).

The following table presents the accidentals together with their associated ratios and cents deviations. To calculate the cents deviation from Equal Temperament of a specific written pitch (if desired) the following shortcut may be used:

1.) Find the cents deviation of the Pythagorean pitch, by calculating how many fifths it is away from A, multiplying by 2, and using a plus sign if it is on the sharp side and a minus if it is on the flat side.

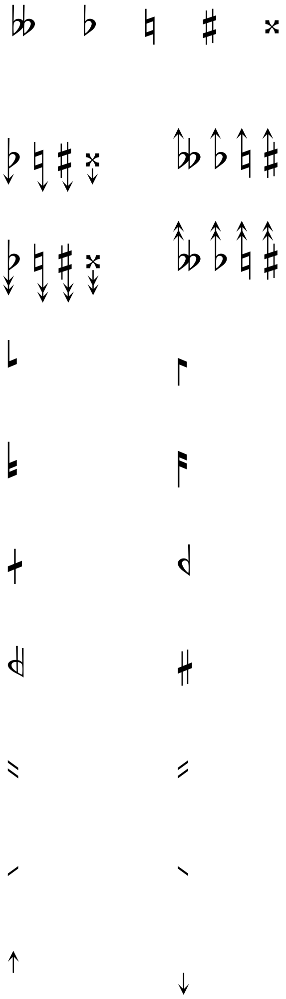
2.) For each microtonal accidental, add or subtract its approximate cents value (as given above), keeping in mind whether the accidental is raising or lowering the pitch.

The resulting value should be a cents deviation within 1 or 2 cents accuracy, which is an acceptable starting point for fine-tuning by ear.

ACCIDENTALS **EXTENDED HELMHOLTZ-ELLIS JI PITCH NOTATION**

for Just Intonation
designed by Marc Sabat and Wolfgang von Schweinitz

The exact intonation of each pitch may be written out by means of the following harmonically-defined signs:



Pythagorean series of fifths – the open strings
(. . . c g d a e . . .)

lowers / raises by a syntonic comma
81 : 80 = ***circa 21.5 cents***

lowers / raises by two syntonic commas
circa 43 cents

lowers / raises by a septimal comma
64 : 63 = ***circa 27.3 cents***

lowers / raises by two septimal commas
circa 54.5 cents

raises / lowers by an 11-limit undecimal quarter-tone
33 : 32 = ***circa 53.3 cents***

lowers / raises by a 13-limit tridecimal third-tone
27 : 26 = ***circa 65.3 cents***

lowers / raises by a 17-limit schisma
256 : 255 = ***circa 6.8 cents***

raises / lowerss by a 19-limit schisma
513 : 512 = ***circa 3.4 cents***

raises / lowers by a 23-limit comma
736 : 729 = ***circa 16.5 cents***

In addition to the harmonic definition of a pitch by means of its accidentals, it is also possible to indicate its absolute pitch-height as a cents-deviation from the respectively indicated chromatic pitch in the 12-tone system of Equal Temperament.

The attached arrows for alteration by a syntonic comma are transcriptions of the notation that Hermann von Helmholtz used in his book "Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik" (1863). The annotated English translation "On the Sensations of Tone as a Physiological Basis for the Theory of Music" (1875/1885) is by Alexander J. Ellis, who refined the definition of pitch within the 12-tone system of Equal Temperament by introducing a division of the octave into 1200 cents. The sign for a septimal comma was devised by Giuseppe Tartini (1692-1770) – the composer, violinist and researcher who first studied the production of difference tones by means of double stops.

VORZEICHEN

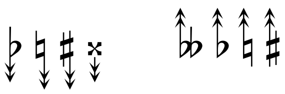
EXTENDED HELMHOLTZ-ELLIS JI PITCH NOTATION

für die natürliche Stimmung
konzipiert von Marc Sabat und Wolfgang von Schweinitz

Die Stimmung jedes Tons ist mit folgenden harmonisch definierten Vorzeichen ausnotiert:

𝄫 ♭ ♮ ♯ 𝄪 — *Pythagoreische Quintenreihe der leeren Streicher-Saiten*
(… c g d a e …)

Erniedrigung / Erhöhung um ein Syntonisches Terzkomma
81 : 80 = ***circa 21.5 cents***



Erniedrigung / Erhöhung um zwei Syntonische Terzkommas
circa 43 cents

Erniedrigung / Erhöhung um ein Septimenkomma
64 : 63 = ***circa 27.3 cents***

Erniedrigung / Erhöhung um zwei Septimenkommas
circa 54.5 cents

Erhöhung / Erniedrigung um den undezimalen Viertelton der 11er-Relation
33 : 32 = ***circa 53.3 cents***

Erniedrigung / Erhöhung um den tridezimalen Drittelton der 13er-Relation
27 : 26 = ***circa 65.3 cents***

Erniedrigung / Erhöhung um ein Siebzehner-Schisma
256 : 255 = ***circa 6.8 cents***

Erhöhung / Erniedrigung um ein Neunzehner-Schisma
513 : 512 = ***circa 3.4 cents***

↑ ↓ — *Erhöhung / Erniedrigung um ein Dreiundzwanziger-Komma*
736 : 729 = ***circa 16.5 cents***

Zusätzlich zu der harmonischen Definition der Tonhöhe durch das Vorzeichen für jeden Ton ist auch der Cents-Wert der Abweichung der gewünschten Stimmung von der Tonhöhe des jeweils bezeichneten chromatischen Tons der gleichstufig temperierten Zwölfton-Skala angegeben.

Die attachierten Pfeile für die Alteration um ein Syntonisches Terzkomma sind eine bloße Transkription der Notation, die Hermann von Helmholtz in seinem Buch "Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik" (1863) verwendet hat. Die kommentierte englische Übersetzung "On the Sensations of Tone as a Physiological Basis for the Theory of Music" (1875/1885) stammt von Alexander J. Ellis, der auch eine enorme Verfeinerung der Tonhöhendefinition innerhalb des Zwölftonsystems der gleichstufig temperierten Stimmung durch die Unterteilung der Oktave in 1200 Cents eingeführt hat. – Das Vorzeichen für die Alteration um ein Septimenkomma wurde von Giuseppe Tartini (1692-1770) erfunden, der als Komponist, Geiger und Wissenschaftler die durch Doppelgriffe erzeugten Differenztöne untersucht hat.