# The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020 | LEGEND | update 6.2023

revised by Marc Sabat and Thomas Nicholson | PLAINSOUND MUSIC EDITION | www.plainsound.org

*in collaboration with Wolfgang von Schweinitz, Catherine Lamb, and M.O. Abbott, building upon the original HEJI notation devised by Marc Sabat and Wolfgang von Schweinitz in the early 2000s*

**PYTHAGOREAN JUST INTONATION | generated by multiplying / dividing any reference frequency by powers of PRIMES 2 and 3 only**

… 𝄫 ♭ 𝄮 ♯ 𝄪 …

notate a series of ***perfect fifths*** above / below a reference — $^{3}/_{2} \approx \pm 702.0$ cents *(ca. 2c wider than tempered)*
*each new accidental therefore represents 7 fifths, or an alteration by one apotome* — $^{2187}/_{2048} \approx \pm 113.7$ cents

Frequency ratios including higher prime numbers (5–47) may be precisely written by adding the following distinct accidental symbols. Custom indications for high primes or arbitrary enharmonic substitutions may be devised **or derived algorithmically** by defining ratio alterations from nearby Pythagorean notes and representing these using symbols (accidentals).

**PTOLEMAIC JUST INTONATION | PRIMES up to 5**

notate the consonant ***just major third*** — $^{5}/_{4} \approx \pm 386.3$ cents *(ca. 14c narrower than tempered)*

𝄫 ♭ 𝄮 ♯ 𝄪 𝄫 ♭ 𝄮 ♯ 𝄪

*alteration of* $^{81}/_{64}$ *by one syntonic comma* — $^{81}/_{80} \approx \pm 21.5$ cents

𝄫 ♭ 𝄮 ♯ 𝄪 𝄫 ♭ 𝄮 ♯ 𝄪

*alteration by two syntonic commas* — $^{81}/_{80} \cdot {}^{81}/_{80} \approx \pm 43.0$ cents

~♯ = ♭ ~♭ = ♯

*alteration by one schisma to notate an exact enharmonic substitution* — $^{32805}/_{32768} \approx \pm 2.0$ cents

**SEPTIMAL JI | PRIME 7**

notate the consonant ***natural seventh*** — $^{7}/_{4} \approx \pm 968.8$ cents *(ca. 31c narrower than tempered)*

𝄭 𝄬

*alteration of* $^{16}/_{9}$ *by one septimal comma* — $^{64}/_{63} \approx \pm 27.3$ cents *(symbols proposed by Giuseppe Tartini)*

𝄮 𝄯

*alteration by two septimal commas* — $^{64}/_{63} \cdot {}^{64}/_{63} \approx \pm 54.5$ cents

**UNDECIMAL | PRIME 11**

notate the ***undecimal semi-augmented fourth*** — $^{11}/_{8} \approx \pm 551.3$ cents *(ca. 51c wider than tempered)*

𝄋 𝄲

*alteration of* $^{4}/_{3}$ *by one undecimal quartertone* — $^{33}/_{32} \approx \pm 53.3$ cents *(Richard H. Stein)*

**TRIDECIMAL | PRIME 13**

notate the ***tridecimal neutral sixth*** — $^{13}/_{8} \approx \pm 840.5$ cents *(ca. 59c narrower than a tempered major sixth)*

𝄳 𝄰

*alteration of* $^{27}/_{16}$ *by one tridecimal thirdtone* — $^{27}/_{26} \approx \pm 65.3$ cents *(Gérard Grisey)*

≈♭ = 𝄋𝄳 ≈♯ = 𝄲𝄰

*combination of* $^{11}/_{13}$ *re-notated enharmonically — alteration by the ratio* $^{352}/_{351} \approx \pm 4.9$ cents

𝄎♮ = 𝄲𝄳 𝄎♮ = 𝄋𝄰

*combination of 11 * 13 re-notated as a single symbol — alteration by the ratio* $^{144}/_{143} \approx \pm 12.1$ cents

**HIGHER PRIMES 17 – 47 ***

*17°, 47° favour a rising spelling — alternate notes/alterations, i.e., $^{4131}/_{4096}$ or $^{48}/_{47}$ may be substituted

𝄐 𝄑

*alteration of* $^{2187}/_{2048}$ *by one 17-limit schisma* — $^{2187}/_{2176} \approx \pm 8.7$ cents

𝄖 𝄗

*alteration of* $^{32}/_{27}$ *by one 19-limit schisma* — $^{513}/_{512} \approx \pm 3.4$ cents

𝄧 𝄏

*alteration of* $^{729}/_{512}$ *by one 23-limit comma* — $^{736}/_{729} \approx \pm 16.5$ cents *(James Tenney / John Cage)*

𝄩 𝄱

*alteration of* $^{16}/_{9}$ *by one 29-limit sixthtone* — $^{261}/_{256} \approx \pm 33.5$ cents

𝄃 𝄍

*alteration of* $^{1}/_{1}$ *by one 31-limit quartertone* — $^{32}/_{31} \approx \pm 55.0$ cents *(Alinaghi Vaziri)*

𝄇 𝄈

*alteration of* $^{9}/_{8}$ *by one 37-limit quartertone* — $^{37}/_{36} \approx \pm 47.4$ cents *(Ivan Wyschnegradsky)*

– +

*alteration of* $^{81}/_{64}$ *by one 41-limit comma* — $^{82}/_{81} \approx \pm 21.2$ cents *(Ben Johnston)*

𝄾 𝄿

*alteration of* $^{4}/_{3}$ *by one 43-limit comma* — $^{129}/_{128} \approx \pm 13.5$ cents

𝄅 𝄆

*alteration of* $^{729}/_{512}$ *by one 47-limit quartertone* — $^{752}/_{729} \approx \pm 53.8$ cents

**CENTS** HEJI accidentals may be combined with an indication of their deviation in cents from equal temperament as read on a tuning meter; ♮A 440 Hz is usually defined to be ±0 cents. If this deviation exceeds ±50 cents, the nearest tempered pitch-class may be added: e.g. 𝄳A (−65 cents from ♮A) could include the annotation ♭A+35 placed above or below its accidental.

**TEMPERED NOTES | may be combined with cents deviations to notate free microtonal pitches**

… 𝄫 ♭ 𝄋 𝄭 𝄲 ♯ 𝄪 …

indicate the respective 24-edo quartertone; show which pitch is assigned as reference (deviation of 0 cents)

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*with dedicated microtonal accidental symbols for primes 5 through 47*

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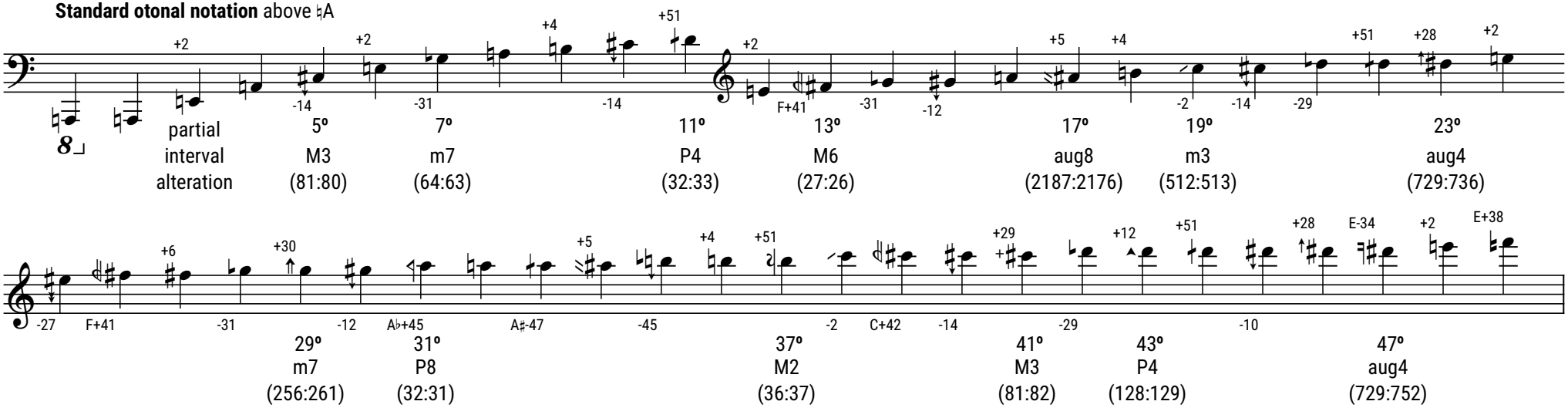
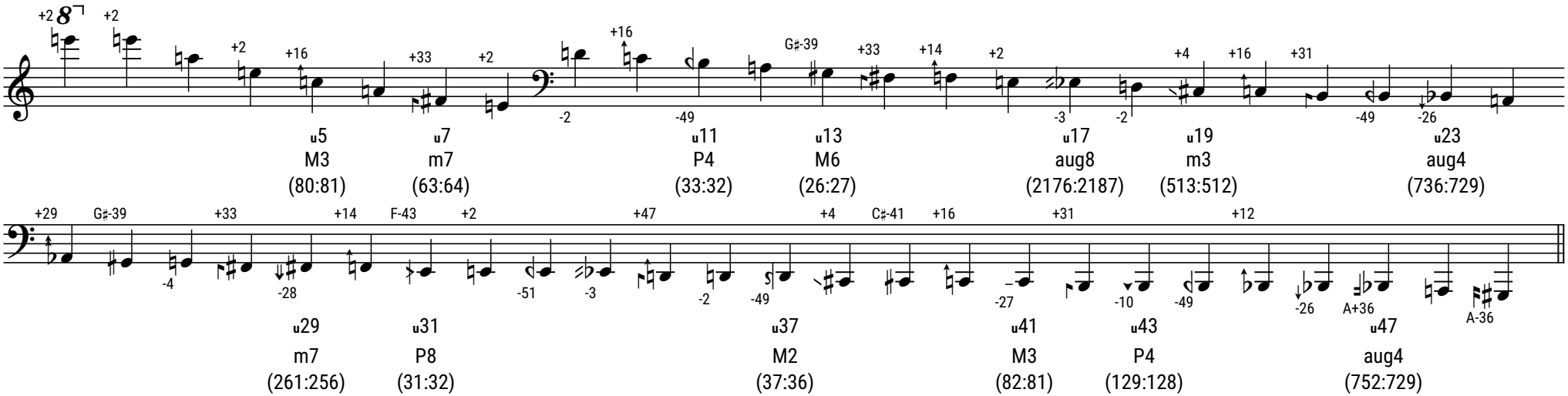
Ratios represent the amount of modification of the Pythagorean notes by each additional symbol, cents indications are deviations that would be shown on a tuning meter with A = 0 cents

**Standard otonal notation** above ♮A




**Standard utonal notation** below ♮E



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𐘱 𐘲
𐘳 𐘴
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𐘷 𐘸
𐘹 𐘺
𐘻 𐘼
𐘽 𐘾
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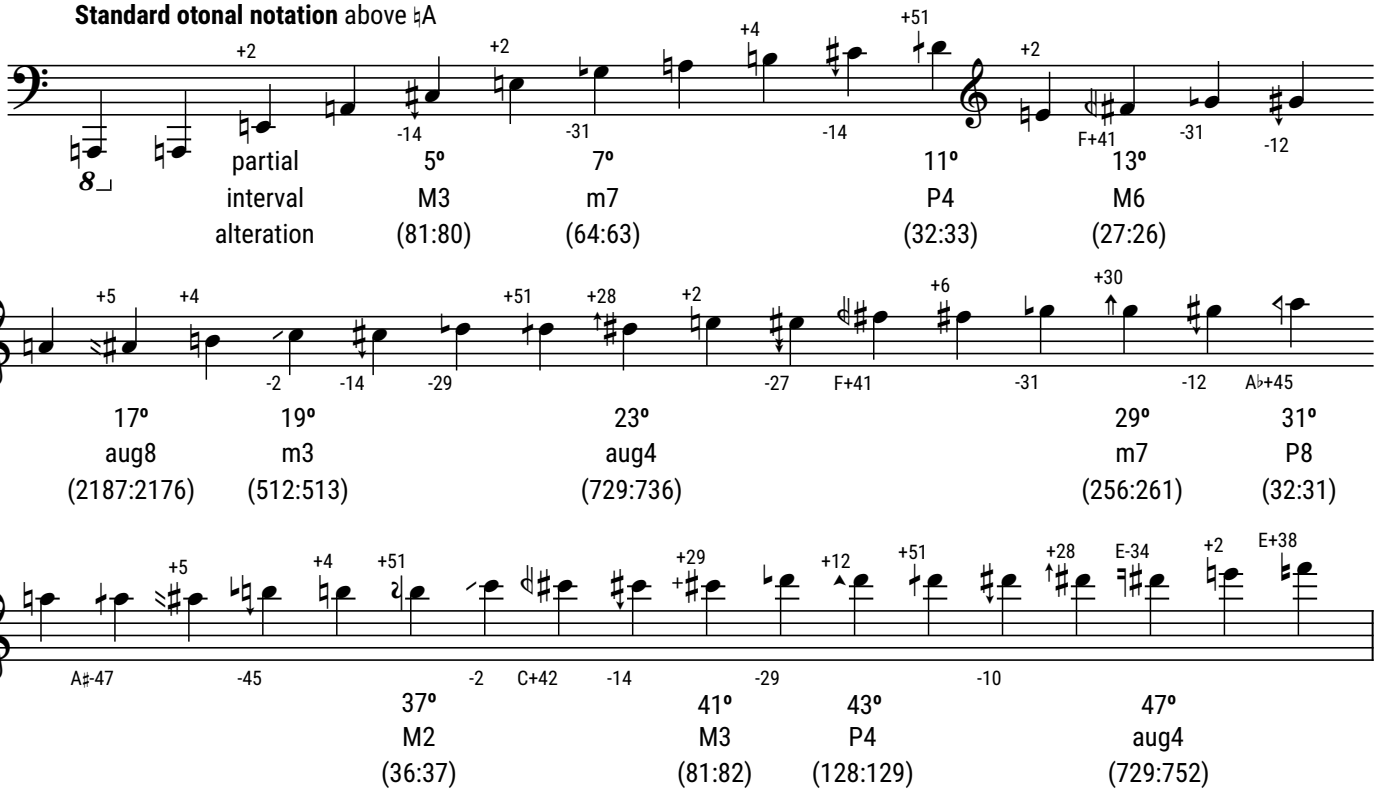


**Standard utonal notation** below ♮E